Restraint Element Deflection Limits

Introduction:

When testing any type of mechanical or structural component, it is necessary to have an adequate definition of failure, and measurable quantity that will allow the point of failure to be accurately determined. The first definition of failure is breakage or fracture of the component. If the component does not break, the failure criterion is usually linked to the yield point of the material of the component. The typically listed yield point of a ferrous material is the ***0.2% Offset Yield Point***. This is the stress level that corresponds to a permanent strain of ***0.002 in/in***.

The purpose of this document is to provide a set of guidelines for predicting the deflection of a component at the ***0.2% Offset Yield Point*** for various generic component types.

Axially Loaded Components:

Figure A6.1.1-1 shows a typical axially loaded component. The component may be either loaded in tension, or compression. The discussion which follows will apply to both tension and compression. If the component is long and slender, and loaded in compression, care must be taken to ensure that the primary failure mode is not in ***buckling***.

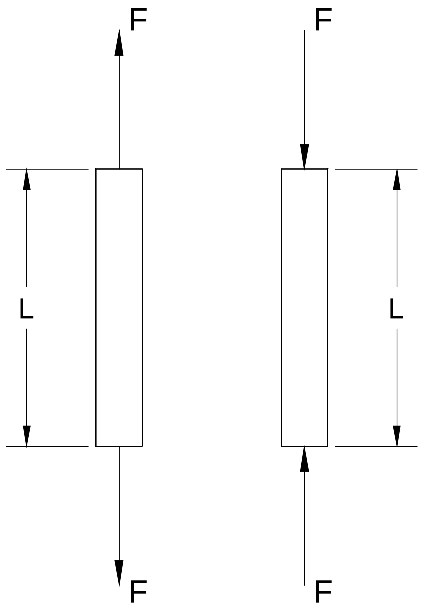

Figure A6.1.1-1: Axially Loaded Components.

For ferrous and other linear elastic materials, the following basic equations will apply.

$$\sigma_A = E * \varepsilon \qquad \text{(Eq. A6.1.1-1)}$$

Where:

E = the elastic modulus of component material, also known as Young's Modulus.
ε = the strain in the component.
σ_A = the axial stress in the component.

The strain acting in the component, ε, is defined as the change in the length, ΔL, of the component under the action of the force F divided by the unloaded length, L, of the component, or:

$$\varepsilon = \Delta L / L$$

(Eq. A6.1.1-2)

Rearrange Equation A6.1.1-1 to solve for ε.

$$\varepsilon = \sigma_A / E$$

(Eq. A6.1.1-3)

Set Equation A6.1.1-2 equal to Equation A6.1.1-3, and solve for ΔL.

$$\Delta L = L * \sigma_A / E$$

(Eq. A6.1.1-4)

Let σ_A be equal to the ***0.2% Offset Yield Point*** stress, σ_{YP}. Then, the deflection for the component at the ***0.2% Offset Yield Point*** is:

$$\Delta L = L * \sigma_{YP} / E$$

(Eq. A6.1.1-5)

Table A6.1.1-1 presents the measurable value of ΔL for various values of L.

Table A6.1.1: ΔL vs. L for Axially Loaded Components.

At a 0.2% Offset Yield Point

Component Length L (in)	Length Change ΔL (in)	Component Length L (in)	Length Change ΔL (in)	Component Length L (in)	Length Change ΔL (in)
1.0	0.0020	16.0	0.0320	60.0	0.1200
2.0	0.0040	18.0	0.0360	72.0	0.1440
4.0	0.0080	24.0	0.0480	84.0	0.1680
6.0	0.0120	30.0	0.0600	96.0	0.1920
8.0	0.0160	36.0	0.0720	108.0	0.2160
10.0	0.0200	42.0	0.0840	120.0	0.2400
12.0	0.0240	48.0	0.0960	132.0	0.2640
14.0	0.0280	54.0	0.1080	148.0	0.2960

Cantilever Component with an End Load:

Figure A6.1.1-2 shows an end loaded cantilever component.

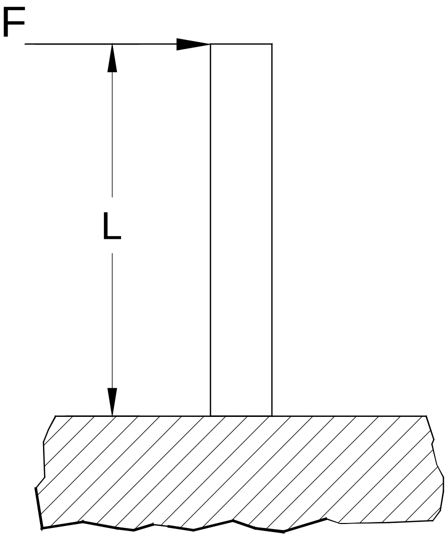

Figure A6.1.1-2: Cantilever Component with an End Load.

Because its impact is insignificant and for the sake of simplicity, the shear stress in the component will be ignored. The maximum deflection for a cantilever beam with an end load occurs at the load point and is found in many standard references as:

$$Y_M = F * L^3 / (3 * E * I) \qquad \textbf{(Eq. A6.1.1-6)}$$

Where:

F = the load applied at the end of the cantilever component.
I = the area moment of inertia of the component parallel to the load.
L = the length of the component.
Y_M = the maximum deflection at the end of the cantilever component.

In general the bending stress in any beam type component is given by:

$$\sigma_B = M * c / I \qquad \textbf{(Eq. A6.1.1-7)}$$

Where:

c = the distance from the neutral axis to the outer fibers of the component.
M = the bending moment in the component at the support.
σ_B = the bending stress in the component at the support.

The bending moment at the support will be as follows.

$$M = F * L \qquad \text{(Eq. A6.1.1-8)}$$

Let σ_{YP} be equal to σ_B and solve for F. This will be the force required to yield the component at the support.

$$F = \sigma_{YP} * I / (L * c) \qquad \text{(Eq. A6.1.1-9)}$$

Substitute Equation A6.1.1-9 into Equation A6.1.1-6 to yield the following result.

$$Y_M = (1/3) * (L^2 / c) *(\sigma_{YP} / E) \qquad \text{(Eq. A6.1.1-10)}$$

The result in Equation A6.1.1-10 will be more useful if it is expressed in a dimensionless form as shown below.

$$(Y_M / L) = (1/3) * (L / c) *(\sigma_{YP} / E) \qquad \text{(Eq. A6.1.1-11)}$$

Table A6.1.1-2 gives the results of Equation A6.1.1-11.

Table A6.1.1-2: *(Y_M / L)* vs. *(L / c)* For a Cantilever Component.

At a 0.2% Offset Yield Point

L/c	Y_M / L	L/c	Y_M / L	L/c	Y_M / L
1.0	0.00067	20.0	0.01333	80.0	0.05333
2.0	0.00133	30.0	0.02000	90.0	0.06000
4.0	0.00267	40.0	0.02667	100.0	0.06667
6.0	0.00400	50.0	0.03333	150.0	0.10000
8.0	0.00533	60.0	0.04000	200.0	0.13333
10.0	0.00667	70.0	0.04667	250.0	0.16667

Simply Supported Component with a Center Load:

Figure A6.1.1-3 shows a simply supported component with a center load. As it is insignificant and for simplicity's sake, the shear stress in the component will be ignored. The maximum deflection for a simply supported beam with a center load occurs at the load point and is found in many standard references as:

$$Y_M = F * L^3 / (48 * E * I) \qquad \text{(Eq. A6.1.1-12)}$$

The maximum bending moment occurs at the center of the beam, and is given by:

$M = F * L / 4$ **(Eq. A6.1.1-13)**

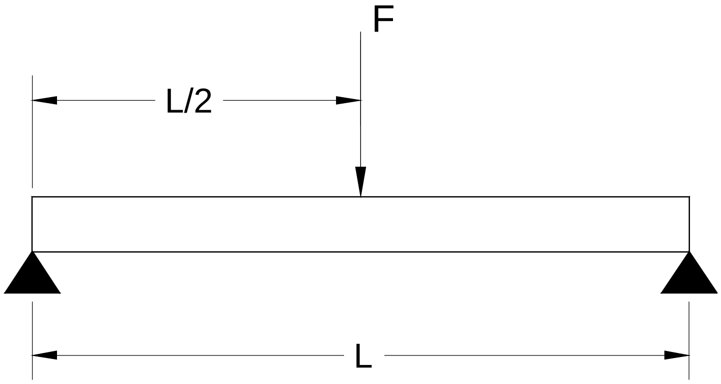

Figure A6.1.1-3: Simply Supported Component with a Center Load.

Following the same procedure as with the cantilever component, we may obtain the following results.

$Y_M = (1/12) * (L^2 / c) *(\sigma_{YP} / E)$ **(Eq. A6.1.1-14)**

The result in Equation A6.1.1-14 will be more useful if it is expressed in a dimensionless form as shown below.

$(Y_M / L) = (1/12) * (L / c) *(\sigma_{YP} / E)$ **(Eq. A6.1.1-15)**

Table A6.1.1-3 gives the results of Equation A6.1.1-15.

Table A6.1.1-3: (Y_M / L) vs. (L / c) For a Simply Supported Component.

At a 0.2% Offset Yield Point

L/c	Y_M / L	L/c	Y_M / L	L/c	Y_M / L
1.0	0.00017	20.0	0.00333	80.0	0.01333
2.0	0.00033	30.0	0.00500	90.0	0.01500
4.0	0.00067	40.0	0.00667	100.0	0.01667
6.0	0.00100	50.0	0.00833	150.0	0.02500
8.0	0.00133	60.0	0.01000	200.0	0.03333
10.0	0.00167	70.0	0.01167	250.0	0.04167

Component with Fixed Supports and a Center Load:

Figure A6.1.1-4 shows a component with fixed ends and a center load. Due to its minimal impact and to simply the analysis, the shear stress in the component will be ignored.

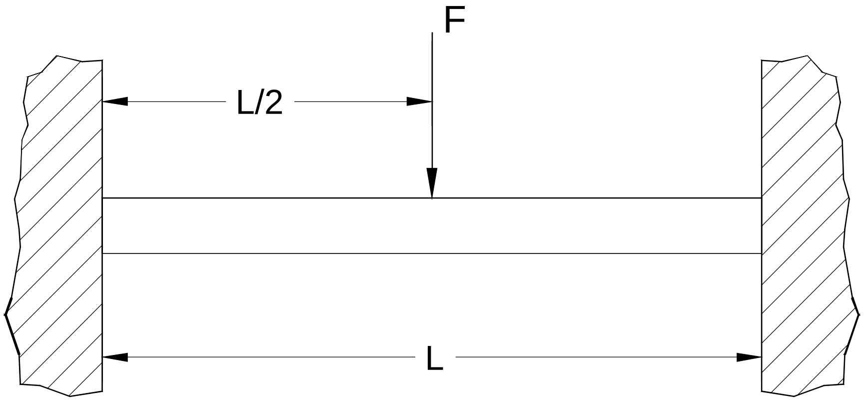

Figure A6.1.1-4: Component with Fixed Ends and a Center Load.

The maximum deflection for a beam with fixed ends and a center load, again, occurs at the load point and is found in many standard references as:

$$Y_M = F * L^3 / (192 * E * I) \qquad \textbf{(Eq. A6.1.1-16)}$$

The maximum bending moment occurs at the center of the beam, and is given by:

$$M = F * L / 8 \qquad \textbf{(Eq. A6.1.1-17)}$$

Following the same procedure as with the simply supported component, we may obtain the following results.

$$Y_M = (1/24) * (L^2 / c) *(\sigma_{YP} / E) \qquad \textbf{(Eq. A6.1.1-18)}$$

The result in Equation A6.1.1-18 will be more useful if it is expressed in a dimensionless form as shown below.

$$(Y_M / L) = (1/24) * (L / c) *(\sigma_{YP} / E) \qquad \textbf{(Eq. A6.1.1-19)}$$

Table A6.1.1-4 gives the results of Equation A6.1.1-19.

Table A6.1.1-4: (Y_M / L) vs. (L / c) For a Component with Fixed Supports

At a 0.2% Offset Yield Point

L/c	Y_M / L	L/c	Y_M / L	L/c	Y_M / L
1.0	0.000083	20.0	0.00167	80.0	0.00667
2.0	0.000167	30.0	0.00250	90.0	0.00750
4.0	0.000333	40.0	0.00333	100.0	0.00833
6.0	0.000500	50.0	0.00417	150.0	0.01250
8.0	0.000667	60.0	0.00500	200.0	0.01667
10.0	0.000833	70.0	0.00583	250.0	0.02083

Summary of Results:

For the axially loaded components, Equation A6.1.1-5 may be written for a ***0.2% Offset Yield Point*** as follows.

$$\Delta L / L = 0.002 \qquad \text{(Eq. A6.1.1-20)}$$

For the components that may be modeled as some type of beam element in bending, the results in Tables A6.1.1-2 through A6.1.1-4 may be best summarized, and be more useful in the graphical form as shown in Figure A6.1.1-5.

Equation A6.1.1-20 and Figure A6.1.1-5 will allow an investigator to predict the deflection of a component at failure as defined by the ***0.2% Offset Yield Stress***. If significant plastic deformation is to be permitted, then this deflection will allow the investigator to predict the deflection at which plastic deformation has truly begun.

Figure A6.1.1-5: Y_M/L vs. L/c at a 0.2% Offset Yield Point For Beam Type Components